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HEAT TRANSFER IN A HORIZONTAL FLUID LAYER HEATED FROM BELOW UPON ROTATION OF ONE OF THE BOUNDARIES

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The mixed convection in a horizontal fluid layer which is generated by uniform heating from below and by rotation of one of the boundaries of the layer was studied experimentally. The region occupied by the fluid is a cylinder of radius 320 mm *and height* 45 mm. *Either the upper or the lower boundary together with the side wall rotates. For Rayleigh numbers* $\mathrm{Ra} \simeq 2 \cdot 10^7$, *in a broad range of Reynolds numbers, based on experimental data we constructed mean-temperature profiles along the normal to the upper boundary and with a uniform step over the radius. In addition, we obtained data on the radial thermal stratification of the fluid, the integral flow through the fluid layer, and information on temperature fluctuations. The complicated character of the dependence of the heat transfer on the Reynolds number was shown. The obtained dependences of the heat transfer and temperature inhomogeneity on Reynolds numbers was explained qualitatively.*

Introduction. A flow that occurs in a horizontal fluid layer heated from below (the Rayleigh–Benard convection) are among the canonical phenomena in treating the problems of hydrodynamic stability and laminar–turbulent transition [1]. It can serve as a model of processes occurring in the atmosphere, the ocean, the Earth's mantle [2, 3], engineering devices, and technological processes [4].

In many cases, buoyancy forces, which are responsible for excitation of the thermogravitational convection, are comparable with forces generating the forced convection. The examples are heat transfer from an underlying surface to a wind-driven atmosphere or from a horizontal surface in an engineering device in the presence of a forced flow. A forced flow is usually initiated by a pressure drop or by rotation of bodies submerged in a fluid or the fluid-bounding surfaces. In the latter case, the flow structure and the heat transfer characteristics are determined by the joint action of buoyancy forces, pressure difference, friction at the walls, and centrifugal and Coriolis forces.

In this paper, we consider the mixed convection in a horizontal fluid layer upon heating from below and rotation of the layer boundaries. The upper boundary (cover) and the lower boundary (bottom) together with the side wall rotate independently. The fluid layer represents a cylinder of height H and radius R_0.

In the case where the boundaries are at rest, the convection is described by dimensionless parameters: the Rayleigh number $\mathrm{Ra} = \beta g \Delta T H^3/(a\nu)$, the Prandtl number $\mathrm{Pr} = \nu/a$, the geometric parameter $\Gamma = R_0/H$, and the conditions at the boundaries. Berdnikov and others [5–7] studied the flow structure for various values of governing parameters. If the cover and the bottom rotate with the same angular velocity ($\Omega_1 = \Omega_2$), the fluid layer rotates as a whole about the vertical axis with velocity Ω_1. In this case, the structure of a convective flow is affected by additional Coriolis and centrifugal forces. The influence of the Coriolis force is determined by a dimensionless criterion, namely, the Taylor number ($\mathrm{Ta} = 4\omega^2 H^4/\nu^2$). If $R_0\Omega^2 \ll g$, the effect of centrifugal forces is negligible [1]. The thermogravitational convection in a rotating fluid layer heated from below has been less studied than the Rayleigh–Benard problem. Some data on the flow structure and heat transfer through a rotating layer of a fluid were presented in [8].

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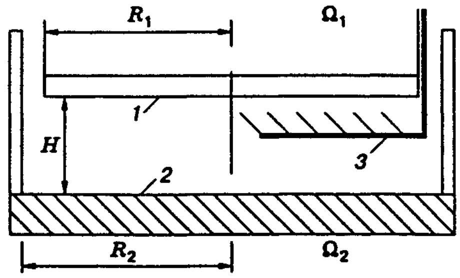

Fig. 1. Layout of the working section and positions of a temperature-sensitive element: 1) upper heat exchanger (cover); 2) lower heat exchanger (bottom); 3) probe for temperature measurements.

When the cover or the bottom rotates, the rotating boundary is a source of forced convection, which is determined by a dimensionless criterion, namely, the Reynolds number: $\mathrm{Re}_1 = \Omega_1 R_0^2/\nu$ or $\mathrm{Re}_2 = \Omega_2 R_0^2/\nu$. In an isothermal fluid, an axisymmetrical flow of scale R_0 [9] appears during rotation of the boundaries, and this flow becomes turbulent with increasing Re. The first experimental results concerning the flow in a horizontal layer of a nonisothermal fluid when one boundary rotates were presented in [10]. In the range of Rayleigh numbers $(1\text{–}8) \cdot 10^7$, one can observe a significant radial inhomogeneity of the temperature field and a local heat flux near the upper rotating boundary as Re_1 increases.

The present study is a continuation of [10] and is aimed at studying the temperature-field structure and heat transfer during rotation of the layer boundaries. Here we give experimental data on the structure of the temperature field in the vicinity of the upper boundary of the layer and on heat transfer for various Reynolds numbers (Re_1 and Re_2) and the Rayleigh numbers $\mathrm{Ra} \approx 2 \cdot 10^7$. The experiments were carried out with the use of ethyl alcohol ($\mathrm{Pr} = 16$ at 20°C).

Description of the Experiment. The setup employed for studying the hydrodynamics and heat exchange in horizontal fluid layers heated from below with rotating boundaries of the layer was described in [10]. The simplified scheme of the working section of the setup is depicted in Fig. 1. The fluid layer to be examined is bounded by an upper transparent heat exchanger 1, a lower heat exchanger 2, and a transparent side wall. The upper heat exchanger has a complicated design; its working surface is a 5 mm-thick mirror glass which, in turn, is the bottom of a thin-walled metal cylinder of diameter 640 mm which rotates about the vertical axis. A fixed transparent heat exchanger through which thermostatic water is pumped is located above the mirror glass. The gap between this heat exchanger and the glass is filled with water. The lower heat exchanger consists of three brass plates of diameter 640 mm and thickness 15 mm. An electrical heater is positioned between two lower plates. The distance between two upper plates is calibrated by glass reference insertions of thickness 4.0 mm. The gap formed by the insertions between the plates was airtight over the circumference and was filled with PÉS-5 silicon-organic oil. The unit consisting of two upper plates and an oil interlayer between them is a gauge of integral heat flux. A stainless steel ring with an electrical security heater inside is pressed against the upper brass plate. Two side vertical transparent cylindrical walls were pressed to the circular security heater. The lower heat exchanger was fixed on a platform which can rotate. To transfer the electrical energy and to apply signals of the gauges and control signals to the mobile units of the setup, current collectors were used. The temperature of the lower boundary of the layer was set and kept constant by a PIT-3 temperature-control device, and a battery of copper–constantan thermocouples served as a feedback gauge. The temperature of the circular security heater was controlled by an independent temperature-control device, which allowed us to eliminate the horizontal temperature gradient on the lower plate because of heat transfer to the side walls.

The temperature of the layer boundaries was measured by copper–constantan thermocouples or batteries with five thermocouples in each. Blind holes were drilled in the brass plates, and the thermocouples and the batteries were glued into them with an epoxy glue with a filler. For temperature measurements on the glass surface, the thermocouples were flush-pasted into the grooves. One thermocouple was placed in the center, and three thermocouples were positioned at a distance of approximately 150 mm from the edge in 90° from each other. In temperature measurements in the rotating system, the second junction of the thermocouple was placed in a thermostat with an electronic control scheme. This thermostat was placed on the same rotating units of the setup. The thermostat temperature was kept constant with an accuracy of ± 0.01°C.

The system of specifying the boundary temperature conditions ensures the accuracy of keeping a temperature of ± 0.04°C at the layer boundaries. In fact, the setup allowed us to perform experiments at independent coaxial uniform rotation of the upper and lower boundaries within the range of angular velocities $5 \cdot 10^{-3}$ to 2 rad/sec with an error not worse than $\pm 1\%$.

To measure the local temperature in a liquid volume, we used a probe consisting of six thermocouples 3 (Fig. 1) made from L-shaped glass tube-holder whose external diameter was 2 mm. Six Nichrome–constantan thermocouples were placed in this tube-holder and were fed through holes in it such that the thermocouple junctions were at the same height (at a distance of 20 mm from the tube-holder), and they were equally distant from each other over the radius, beginning with the center $r = 0$, with a step of approximately 50 mm. The system of motion of the probe over the normal to the upper horizontal surface was mounted on the carrying unit of this layer boundary. The carrying unit consists of a microscrew, a stepping engine, a control unit, and a clock-type indicator to control the motion of the probe, with a 0.01 mm value of scale division. The control unit for the stepping engine makes it possible to control the motion of the probe under the conditions of a laboratory system and, if necessary, by means of a computer. Two operational modes were envisaged: the displacement step Δz is equal 0.05 mm in the first mode and 0.5 mm in the second. The signals from the thermocouples were applied to a commutator and were transmitted to the laboratory system through current collectors.

In the first series of experiments, we examined the structure of the temperature field near the upper heat exchanger ($z = 0$–7 mm). The setup reached a stationary state, which is characterized by the constant temperature difference between the bottom and the cover ΔT and the constant angular velocities of the cover and the bottom (Ω_1 and Ω_2, respectively). The state of the liquid layer was characterized by dimensionless parameters, namely, the Rayleigh numbers and the Reynolds numbers (Re_1 and Re_2). During data acquisition on the temperature field, the setup was in a steady state of heat exchange. The signals were first recorded in the initial position of the probe, when the junctions of all the thermocouples touched the upper bound ($z = 0$), and subsequent measurements were carried out after a displacement at a given step, etc. The recording time of each cycle ranged from 4 to 20 min and depended on the mode (Ra and Re) and the position of the thermocouple. Measurement results were processed on a microcomputer with the use of reliable procedures for statistical processing of occasional signals [11].

In the second series of experiments, we examined the integral heat flux and the structure of the temperature field near the cover outside the thermal boundary layer ($z = 7$ mm) for various values of the Reynolds number. The setup was brought into the initial steady state which is characterized by the Ra and Re values. Measurements necessary for calculation of the Ra, Re, and Nu values were carried out, and the results of signals from all the thermocouples were recorded. After that, the velocity of rotation of the cover or the bottom was changed, and the entire cycle of measurements was repeated in 20–60 min. The exposure period was estimated experimentally, depending on the time of reaching a steady-state regime of heat exchange.

Temperature Field near the Upper Boundary. In the initial, for this problem, state (the Rayleigh–Benard steady-state turbulent convection), the radial temperature "stratification" is absent, which indicates the isotropy in the horizontal plane of the spatial flow structure and the statistical characteristics of the temperature and velocity fields.

The temperature-field structure in the vicinity of the upper bound changes considerably as the upper or the lower boundary of the layer rotates. Figure 2 shows the mean temperature versus the distance to the upper boundary for $\Omega_1 = 0.6$ rad/sec, which corresponds to $\mathrm{Re}_1 = 4.7 \cdot 10^4$ for $\Omega_2 = 0$ as well. Mean-temperature profiles were constructed for six values of the radii. Clearly, the temperature stratification over the radius is

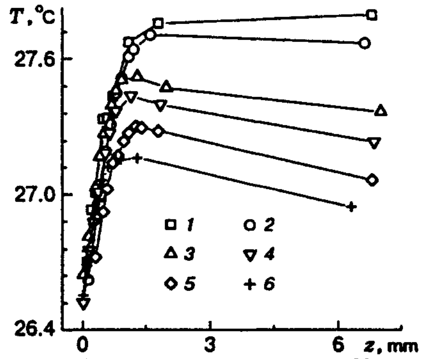

Fig. 2. Mean-temperature profiles at the upper boundary of the layer: $Ra = 2.28 \cdot 10^7$, $Re_1 = 4.43 \cdot 10^4$, and $Re_2 = 0$; points 1–6 refer to $r = 2$, 53, 103, 147, 200, and 242 mm, respectively.

observed near the upper layer boundary outside the thermal boundary layer: for $z > 1$ mm, the temperature on the axis is higher than at the cover's edge. The temperature changes slightly in the axial direction in the core of the liquid layer.

If one increases the velocity of rotation of the bottom (the direction of this rotation coincides with that of the rotation of the cover) at a constant velocity of the cover, the structure of the temperature field will change. With increase in Ω_2, the temperature inhomogeneity over the radius decreases and disappears as the velocity of rotation of the upper boundary ($\Omega_2 = \Omega_1$) is reached. This operating condition corresponds to the thermogravitational convection in a rotating layer of a fluid [8]. If the direction of rotation of the bottom is opposite to that of the cover, the temperature stratification in the vicinity of the upper boundary behaves in a complicated manner, depending on the values of Ω_1 and Ω_2. For example, for $Re_1 > 5 \cdot 10^4$ and $Re_2 > 2 \cdot 10^4$, the stratification is almost negligible.

Rotation of the cover and the bottom exerts a great effect not only on the mean characteristics of the temperature and velocity fields, but also on their statistical characteristics. As was shown in [10], rotation of only the cover or only the bottom decreases markedly the temperature dispersion in the entire liquid volume. With the cover and the bottom rotated simultaneously, it depends on the velocity of rotation of the boundaries, their mutual direction, and the coordinates of the point of observation. For example, when the upper boundary rotates with angular velocity $\Omega_1 = 0.6$ rad/sec and the lower rotates in the opposite direction with angular velocity $\Omega_2 = 0.16$ rad/sec, the radial dependence of the temperature dispersion is observed. The amplitude of temperature fluctuations is maximum near the axis of rotation and decreases as the radius increases. At large distances from the axis of rotation ($r > 150$ mm), the dispersion is small, and it is almost independent of r. For $r < 150$ mm, the maxima are observed on the dependences $\sigma = f(z)$: the amplitude of temperature fluctuations is maximum at a distance of 0.5–0.8 mm from the upper boundary. With increase in the velocity of rotation of the cover, the spread over the radius decreases. The dispersion does not decreases only on the axis of rotation, which indicates the presence of oscillations in the ascending jet alone.

Heat Transfer and Temperature Stratification. Experimental results obtained for the temperature-field structure in the vicinity of the cover and the integral heat flux are shown as the dimensionless temperatures T_i^* and the modified Nusselt number Nu^* versus the Reynolds number found on the basis of the velocity of rotation of the cover or the bottom. The subscript i in the mean temperature corresponds to the number of the thermocouple positioned on the radius r_i ($i = 1, \ldots, 6$). Here $T_i^* = (T_i - T_1)/(T_2 - T_1)$, $Nu^* = Nu/Ra^{1/3}$, and $Nu = qH/\lambda(T_2 - T_1)$ (q is the heat-flux density averaged over the time and the working-surface area). For various Ω_1 and Ω_2, in steady-state operating modes, the temperature drops differed somewhat from each other, and,hence, the Ra values. To present results of the integral heat flux measurements in a universal form, it is convenient to introduce a modified Nusselt number $Nu^* = Nu/Ra^{1/3}$. For brevity,

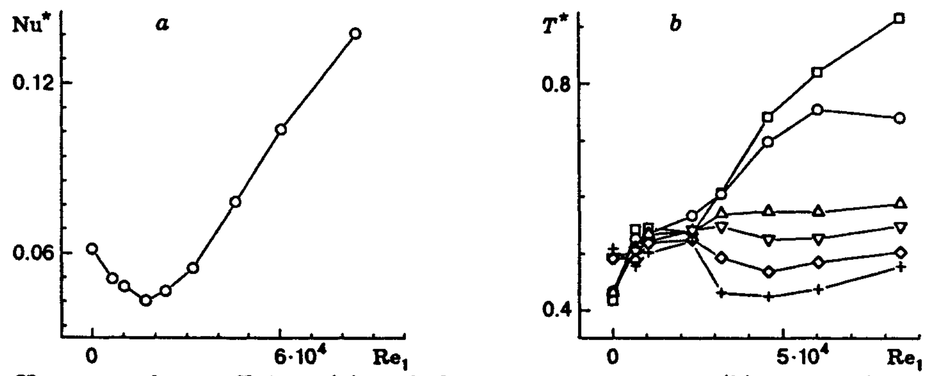

Fig. 3. Heat-transfer coefficient (a) and the mean temperature (b) versus the Reynolds number (for the rotating cover): Ra = (1.5–2.4) $\cdot 10^7$ and $Re_2 = 0$; the notation is the same as in Fig. 2.

we shall call this number, i.e., the normalized dimensionless heat-transfer coefficient as a matter of fact, a heat-transfer coefficient. The range of variation of the values of the Rayleigh number is given in the legends.

Figure 3 shows the heat-transfer coefficient in the case of heat transfer through a fluid layer and the dimensionless mean temperature for six radii near the upper boundary ($z = 7$ mm) versus the velocity of rotation of the cover. Similar dependences on the velocity of rotation of the bottom with the fixed cover are shown in Fig. 4.

The dependences $Nu^*(Re_1)$ and $Nu^*(Re_2)$ behave similarly: for $Re < 1.5 \cdot 10^4$, a decrease in heat transfer is first observed as the velocity of rotation increases. For higher velocities which correspond to $Re > 4 \cdot 10^4$, the heat transfer intensifies. We shall denote the Reynolds numbers at which the dependences indicated above have a minimum by Re_{1cr} and Re_{2cr}. The values of these quantities found from Figs. 3a and 4a differ from each other: $Re_{1cr} = 2 \cdot 10^4$ and $Re_{2cr} = 3 \cdot 10^4$. This is connected with that, in the first case, the forced flow is caused by rotation of the upper boundary alone, and, in the second, a flow is formed when the lower boundary rotates together with the side wall. For $Re > 3 \cdot 10^4$, the almost linear increase in heat transfer is observed. The tangent of the slope angle for the dependence $Nu^*(Re_1)$ is approximately 2.5 times larger than for the dependence $Nu^*(Re_2)$. For small Ω, the decrease in heat transfer through the fluid layer for $Re < 1.5 \cdot 10^4$ is first caused by the laminating action of a shear flow generated by the rotating cover or bottom on the Rayleigh–Benard turbulent convection and then by the laminating action of fluid rotation in the core. Since the thermogravitational convection exerts a dominant effect in a turbulent regime, which is observed for $Re < 1.5 \cdot 10^4$, the main contribution to heat transfer comes from the convective or the fluctuating component $Q_{conv} = (T'v')_{av}$. While rotating the boundary, the displacement of a small volume of the fluid, which is heated up at the second boundary and is hot relative to the ambient medium, in a shear field leads to its expansion and a faster equalization of its temperature with the temperature of the ambient medium. Observations of the spatial flow structure at the upper layer boundary have shown that when the rotation first begins at the periphery ($r \approx R_0$) and then approaches the center of the layer, the boundary layer laminates, and a small-scale thermogravitational-in-nature vortex flow disappears [12]. This causes a decrease in the amplitude of temperature fluctuations and, hence, the convective component of the heat flux in the heat transfer. The increase in heat transfer for $Re > 4 \cdot 10^4$ is due to the formation of a meridional flow of centrifugal nature with scale R_0 under the rotating surface and a transition to the turbulence of hydrodynamic nature in the boundary layers on the heat-transfer surfaces [9].

The temperature stratification in the neighborhood of the upper boundary becomes pronounced for $Re > 3 \cdot 10^4$ (Figs. 3b and 4b). If only the cover and only the bottom rotate, one can observe temperature stratifications of opposite signs. Thermal stratification can be regarded as a characteristic that offers information on the flow structure. For example, the cover rotates with velocities at which $Re_1 > 4 \cdot 10^4$, and a circulating flow of scale R_0 is established: an ascending jet is formed in the center of the flow, near the cover the flow has the radial velocity component directed from the center to the edge of the cover, a

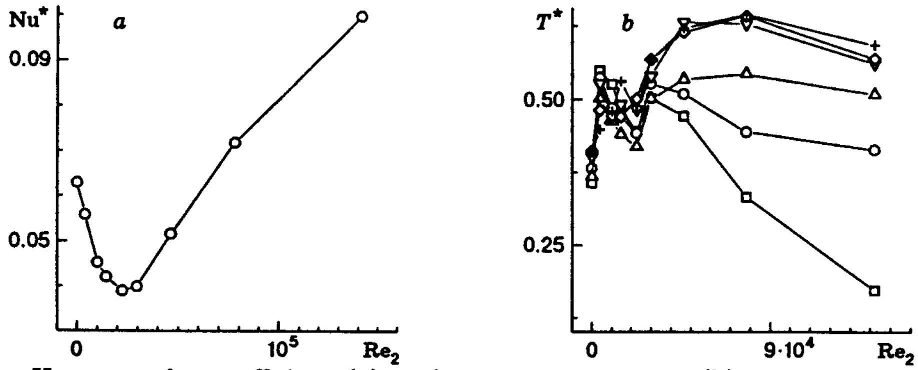

Fig. 4. Heat-transfer coefficient (a) and mean temperature (b) versus the Reynolds number (for the rotating bottom): $Ra = (1.7\text{–}2.4) \cdot 10^7$ and $Re_1 = 0$; the notation is the same as in Fig. 2.

descending flow is formed along the side wall, and the radial velocity component in the vicinity of the bottom is directed to the center. In this flow pattern, the ascending jet at the center has a temperature close to the bottom temperature. As the fluid approaches the cover's edge, it becomes colder, and its temperature decreases monotonically (see Fig. 3b). If the bottom rotates with a velocity at which $Re_2 > 4 \cdot 10^4$, a circulating flow of inverse direction is formed. In this case, an ascending flow of cold fluid is formed in the center, and an ascending flow of a hot fluid is formed at the periphery of the layer. The temperature stratification $\delta T^*_{1.6} = T^*_1 - T^*_6$ increases with increase of Re. For the same values of Reynolds numbers, the magnitude of the horizontal temperature stratification of the fluid core is larger for the case of cover rotation: $\delta T^*_{1.6} = 0.38$ for $Re_1 = 6 \cdot 10^4$ and $\delta T^*_{1.6} = 0.2$ for $Re_2 = 6 \cdot 10^4$. It is necessary to note the different character of temperature stratification near the upper boundary when only the cover or the bottom rotates. During rotation of the bottom the temperature difference on the radii, which is equal to 147, 200, and 242 mm, becomes significant only for $Re > 1.4 \cdot 10^5$, and when the cover rotates, the difference between the temperatures on these radii is marked already for $Re_1 = 3 \cdot 10^4$ and $\delta T^*_{4.5.6} = 0.1$ and varies little with increase of the Reynolds number.

Conclusions. The present study of the Rayleigh–Benard turbulent thermogravitational convection in an immobile fluid layer supplements the results of [5–7], where a fluid with $Pr = 16$ for $Ra \approx 2 \cdot 10^7$ and the ratio of the horizontal size of the layer to the height equal to 6 was investigated. Here this ratio is equal to 13. The problem of the effect of the side boundaries and the relative size of the horizontal layer ($2R_0/H$) remains open in a turbulent flow regime as well, because the existence of a large-scale nonstationary cellular structure was shown in [12]. The boundedness of the layer causes the change in the behavior (the irregular chaotic drift in the horizontal plane) of a cellular flow and can lead to a change of the statistical characteristics of the velocity and temperature fields. The more than twofold increase in the horizontal size of the layer did not affect the spatial form of the flow and the statistical characteristics of the temperature field.

The isotropy of all the quantities being measured in the radial direction is characteristic of the turbulent thermogravitational convection in immobile and uniformly rotating fluid layers. The temperature field has a boundary-layer character in the entire range of governing parameters. The mean temperature depends slightly on z behind the boundary layer. The thickness of the boundary layer increases with increase of the velocity of rotation of the fluid layer as a whole (the Taylor number). The variance of temperature fluctuations decreases as the angular velocity increases.

When the upper or the lower boundary rotates, one can observe the radial inhomogeneity (the stratification) of the mean temperature in the fluid core. The magnitude of the thermal stratification depends on the angular velocity of rotation of the boundary. The stratification is caused by the appearance of a large-scale meridional flow because of centrifugal forces. The sign of the stratification depends on the direction of the global circulating flow. If the fluid under the upper boundary moves from the center to the edge, the temperature in the center is higher than at the edge; when the fluid moves in the opposite direction, the sign of the stratification becomes opposite.

The dependences of the dimensionless heat-transfer coefficient on the velocity of rotation of the upper or the lower boundary while the other is immobile behave similarly. For low velocities of rotation, which correspond to $\mathrm{Re} < 1.5 \cdot 10^4$, a decrease in heat transfer is observed, and the heat transfer becomes more intense for high velocities ($\mathrm{Re} > 4 \cdot 10^4$). The dependences $\mathrm{Nu}^*(\mathrm{Re}_1)$ and $\mathrm{Nu}^*(\mathrm{Re}_2)$ reach a minimum at critical velocities of rotation. The values of the critical Reynolds numbers differ from each other ($\mathrm{Re}_{1\mathrm{cr}} = 2 \cdot 10^4$ and $\mathrm{Re}_{2\mathrm{cr}} = 3 \cdot 10^4$) and depend on the values of the Ra numbers; the critical value is estimated to be $\mathrm{Gr}/\mathrm{Re}^2 = \mathrm{Ra} \cdot \mathrm{Pr}/\mathrm{Re}^2 \approx 0.8$. Thus, the rotation of the boundary first causes laminarization of the turbulent gravitational convection, leading to a decrease in the intensity of heat transfer through the fluid layer. The laminarization near the upper boundary begins at the periphery of the layer, where the linear velocity of motion of the boundary is maximum. With increase in the velocity of rotation, the laminarization of the boundary layer moves rapidly to the center. With further increase in the velocity of rotation, after the critical value $\mathrm{Re}_{\mathrm{cr}}$ is reached, the contribution of the forced flow increases, and a monotone increase of the heat-transfer intensity is observed as the velocity of fluid circulation in the meridional cross section and turbulence of this flow increase.

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